Notes on the Radix Method of Calculating Logarithms (Continued)
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designed to test the ingenuity of university students. But in these schools, let us aim at developing in our pupils the power to exercise choice of method, and let us widen the range as much as is practicable.

Allow me to conclude by a quotation from a book recently published in America, entitled, *The Teaching of Geometry*, and written by David Eugene Smith:

"The efforts usually made to improve the spirit of Euclid are trivial. They ordinarily relate to some commonplace change of sequence, to some slight change in language, or to some narrow line of applications. Such attempts require no particular thought and yield no very noticeable result. But there is a possibility, remote though it may be at present, that a geometry will be developed that will be as serious as Euclid's, and as effective in the education of the thinking individual. If so, it seems probable that it will not be based upon the congruence of triangles, by which so many propositions of Euclid are proved, but upon certain postulates of motion. . . . If to the postulate of parallel translation we join the two postulates of rotation about an axis, leading to axial symmetry; and rotation about a point, leading to symmetry with respect to a centre, we have a group of three motions upon which it is possible to base an extensive and rigid geometry. It will be through some such effort as this, rather than through the weakening of the Euclid-Legendre style of geometry, that any improvement is likely to come. . . . At present the important work for teachers is to vitalize the geometry they have, . . . seeing to it that geometry is not reduced to mere froth, and recognizing the possibility of another geometry that may sometime replace it,—a geometry as rigid, as thought-compelling, as logical, and as truly educational." W. J. Dobbs.

NOTES ON THE RADIX METHOD OF CALCULATING LOGARITHMS.

(*Continued from* p. 150.)

A somewhat obvious simplification of Briggs' logarithmic process was discovered and given as one of three methods by Robert Flower in a rare small quarto tract, *The Radix a new way of making Logarithms*, published in London by J. Beecroft in 1771. Several tables of radices are given, the largest extending from $r=1$ to 9 and n from 1 to 12 to twenty-three places.

Flower divides the given number, if necessary, by a power of ten and a single digit, so as to reduce the first figure to ·9, and then multiplies by a succession of radices until all the digits become nines. The complement of the sum of the logarithms of the radices to the logarithm of the divisor gives the required logarithm. In some cases it is more convenient to multiply than to divide by a digit, the logarithm of the digit is then added to those of the radices and the complement to 1 taken.

Thus, Ex. iii., Flower finds log 3·9784; divide by 4.

3·9784	(log 4) =	+ 0·6020 5999
0·9946	(log 1·005) =	− 21 6666
0·9995 73	(log 1·0004) =	− 1 7368
0·9999 7283	(log 1·00002717) =	− 869
		− 304
		− 4
		− 3
	log 3·9784 =	0·5997 0845

The figures in brackets are not given by Flower.

Again, to find $\log \pi$, Ex. vii., divide by 4 and multiply the quotient up to 1, subtract the logarithms of the radices from log 4.

This method was rediscovered and published as new by Hearn, 1847; it is generally called after him.

Each radix in general consists of 1 followed by as many ciphers as there are nines already obtained, and the complement to nines of the next one, two, or three digits. Hence they can be determined by simple inspection, and the somewhat tedious divisions are replaced by easy multiplications. When half the digits are nines, the complements of the remainder to nines give the required radices directly, and may be multiplied by μ.

It is occasionally necessary, especially when two or three digit radices are used, ' to force ' or add one to the complement. This is so easily done by simple addition that it is hardly worth while to give a formula. Suppose c is the complement to nines of a period which follows nines, and d, e, are the next periods, the latter of which may almost always be neglected, the period becomes nines if

$$d+\frac{e}{10^n} \geqq \frac{c(c+1)}{10^n}.$$

Thus to find the smallest value of d which will make the second period nines in the case of $\cdot 999\,888$,

$$c=111, \quad d=111\times 112\times 10^{-3}=012\,432,$$

$$0\cdot 999\,888\,012\,432\times 1\cdot 000\,111=0\cdot 999\,999\,000\,001.$$

The method seems to have been very rapidly forgotten, and is dismissed by Hutton in a few words without description. It was revived in a most inconvenient form in *Phil. Trans.*, 1806, by Manning, who used negative radices, in which r was always one, so that a very tedious series of simple subtractions was necessary.

A very full account of the history of the method, with valuable criticisms, is given by A. J. Ellis, *Proc. R.S.*, xxxi. 398 and xxxii. 377, 1881. He attributes most of the modifications which have been subsequently proposed to George Atwood, of "machine" fame, in *An Essay on the Arithmetic of Factors*, printed, possibly for private circulation, by T. Cadell, London, 1786. "Hence it appears that Atwood rediscovered Flower's method, but transformed it in the manner carried out ninety years later, 1876, by Hoppe, and not only anticipated Weddle's method, 1845, but showed the connection of the two methods as that of multiplying the reduced number up to 1 in the first case, and down to 1 in the second."

Atwood gives positive and negative natural radices only in a new form to thirteen decimal places. From (i) $\log(1\pm x)$ is equal to $\pm x$—a correction which is tabulated by Atwood,

$$\log(1\pm\cdot 01)=\pm\cdot 01-\cdot 0^4 5\pm\cdot 0^6\dot{3}-\cdot 0^8 25\pm.$$

Hence the tabular positive correction to be subtracted from $+\cdot 01$ is $-\cdot 00004\,96691\,468$, and the tabular negative correction to be added to $-\cdot 01$ is $-\cdot 00005\,03358\,535$. These corrections may be dealt with separately, and their sum added or subtracted at the end of the operations.

The use of potential radices of the form $\left(1+\frac{1}{10^n}\right)^r$, where r varies from 1 to 9, has been advocated by Orchard, 1848, and Oliver Byrne, 1849, but Ellis remarks, "Although from a potential radix the logarithm of a number can be obtained with the same accuracy as from a numerical radix, yet the process is much longer with the former; and hence it appears that the real use of the potential is to calculate the numerical radix." With much deference to Mr. Ellis' opinion, it seems very doubtful if it is not more simple to calculate the numerical radices directly.

There is also much difference of opinion as to the utility of negative radices. If both kinds of radices are given, some few figures may be saved, but with greatly increased risk of errors of sign. It is not very practical to obtain a number from its logarithm by negative radices, since straightforward working gives the reciprocal and not the number. Hence it is usual to tabulate the complements of the logarithms of the radices and to take the complement of the given logarithm.

It may be of interest to apply the logarithmic and anti-logarithmic processes to the prime 8291, the error in each case being in the eleventh figure.

8291 × 13	Comp. log 88605 66476 9
107783 × 1·0$\bar{7}$	3151 70514 5
10023819 × 1·00$\bar{2}$	86 94587 1
10003771362 × 1·000$\bar{3}$	13 03078 9
100007702305914 × 1·0000$\bar{7}$	3 04016 8
7017667 × 1·00000$\bar{7}$	30401 7
1000000017618 × μ	76 5
log 8291	·91860 69151 4

8291/9 =	log 9	·95424 25094 393
92122222222 × 1·08	log	03342 37554 869497
99492 × 1·005		216 60617 5650
9998946 × 1·0001		4 34272 7686
99999 458946		23497 676
Comp. 541054		03563 55942 879
log 8291		·91860 69151 5

– 91860 691514		99999 55778 5 × 1·000$\bar{8}$
– 08139 308486		7 99996 4
– 08042 190762	83	99991 55782 1 × 1·00$\bar{1}$
– 47 117724		99 99155 8
– 43 451177	1·00$\bar{1}$	99891 56636 3 × 83
– 3 666547		8290·9 99999 8
– 3 474495	1·000$\bar{8}$	
– 192052 × 1/μ		
1 – 442215 = 557785		

·91860 691515		1·00000 66069 0
91381 385238	82	8 00005 3
479 306277		1·00008 66074 3
432 137378	1·01	100 00866 1
47 168899		1·00108 66940 4
43 407748	1·001	1001 08669 4
3 761151		1·01109 75609 8 × 82
3 474217	1·00008	202 219 51219 6
286934 × 1/μ = 660690		8088 780 48784
		8291·000 00003 6

On the whole, for general use, no improvement seems to have been made on the original method of Briggs as modified by Flower; but the work is shortened by tables of radices to two or three digits, if multiplication tables are available.

The general neglect of such a simple and convenient method may have been due to the idea that Briggs' radix method required the use of large and expensive tables, which is far from being the case.

Flower's pamphlet is very scarce and very confusing. He possibly calculated the logarithms of the radices by the aid of a table of the successive square roots of ten, and speaks almost as though he considered the nine digits to be roots of ten. He also mixed up the difficulty he found in calculating the logarithms of the radices with the much more simple matter of using them when found.

Atwood's pamphlet is still more scarce; there are copies in the libraries of the Royal and Royal Astronomical Societies, but not in the British Museum; since it deals with natural logarithms only, it would be of little use to practical computers. It seems worthy of reprinting.

A copy of Flower's tract came into the hands of Leonelli, who published his *Supplément Logarithmique* at Bordeaux in 1802, and an edition was published at Dresden by Leonhardi in 1806. He gave tables of natural and common positive radices to twenty places, and also a table of common two-figure radices to fifteen places. Gray claimed as new a two-figure negative table, 1846, and a two-figure positive table, 1848. The *Supplément* was also so scarce as to be almost unknown until Hoüel reprinted the tables of radices in 1858 and 1866, and the whole work in 1876. Schrön also reprinted the positive radices to sixteen places in 1859.

Perhaps the most convenient and powerful tables of three-figure radices are those of Gray, 1876, to twenty-four places. The one-figure positive and negative tables of Thoman to twenty-seven places, Paris, 1867, seem to be out of print and difficult to procure second-hand.

SYDNEY LUPTON.

THE SIMPLE PENDULUM.

THE current elementary discussion of the Simple Pendulum is unsatisfactory, in that the problem is not reduced with sufficient directness to a case of S.H.M. It has to be artificially prefaced by consideration of a curvilinear motion in which the tangential resolute of the acceleration is proportional in magnitude to the arcual distance from a point of the path; and the discussion of this motion raises new points of difficulty altogether out of proportion to its significance in this connection. The net result is that the student's appreciation of this first of the important applications of S.H.M. to "small oscillations" is lost, in dismay at finding that, even after he has mastered the essential difficulties of the S.H.M. itself, the first good application of it brings yet another awkward hurdle. And the physicist or engineer may well make this another case for railing impatiently at the devices for "dodging the Calculus" by which elementary theory is so apt to be obscured. Nevertheless, the teacher of Mathematics knows how important it is to postpone such Calculus difficulties as, *e.g.*, to pave the way to the complete primitive of the S.H.M. differential equation and its uses (one of which gives the only clear-cut way of handling the ordinary discussion of the Simple Pendulum). But the postponement must not be obtained at the disproportionate cost of artificial complications which, for the sake of "elementary" treatment, cast a fog round the important features of the argument, without contributing anything of independent value to the student's store of knowledge.

The elementary treatment of S.H.M. is well worth preserving. The relation