The Frequency of Transverse Vibrations of a Stretched Indiarubber Cord

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If the fault is on the A side of C, it is necessary to substitute $\frac{mf}{m+f}$ for m. If the fault is on the B side, $\frac{nf}{n+f}$ should be substituted for n. By evaluating the above expression for G, almost any case can be examined, either for the purposes of the present test, or for any application of the guard-wire. The best way to determine the magnitude of possible errors is first to evaluate the quantity included between the square brackets, and then to compare this with $(r+g+b)$.

IX. *The Frequency of Transverse Vibrations of a Stretched Indiarubber Cord.* *By* T. J. Baker, *B.Sc.**

The writer has observed that the pitch of the note produced by twanging an indiarubber band which is stretched over the fingers is influenced but little by the tension, particularly if this is considerable.

The matter appeared worthy of further investigation; and for this purpose a short piece of indiarubber cord, of square section, was attached at one end to the middle of a tambourine-diaphragm, while the other end was bound to a piece of twine which passed over a pulley-wheel and supported a weighed scale-pan.

After placing a weight in the pan, the rubber was allowed to stretch for some minutes, and then the end nearest the pan was firmly clamped. The intensity of the note produced by transverse vibrations of the cord was rendered sufficiently great by the tambourine to permit a determination of pitch by comparison with a sonometer and tuning-fork.

The sectional area of the cord under the various tensions employed was determined by measurements of its two diameters at three different points in its length.

For this purpose a micrometer-gauge reading to ·001 cm. was employed. The observations were continued until the

* Read January 26, 1900. (Communicated by Prof. Poynting.)

cord broke. This happened when its length had increased six-fold. The 38 sets of values thus obtained were plotted on squared paper, and three curves were drawn showing the relation between—

(i.) Length and tension.
(ii.) Length and sectional area.
(iii.) Length and frequency.

The smoothing-out of slight experimental irregularities which these curves effect was found of advantage in the calculations hereafter referred to.

Inspection of the curve connecting length with tension reveals that the relation is strictly linear between those points at which the length of the cord becomes respectively $2\frac{1}{2}$ and 5 times its natural length.

Untrustworthy readings were obtained with further stretching because an increase of tension did not produce its full effect until a very considerable time had elapsed. With the smaller stretchings this difficulty was not encountered.

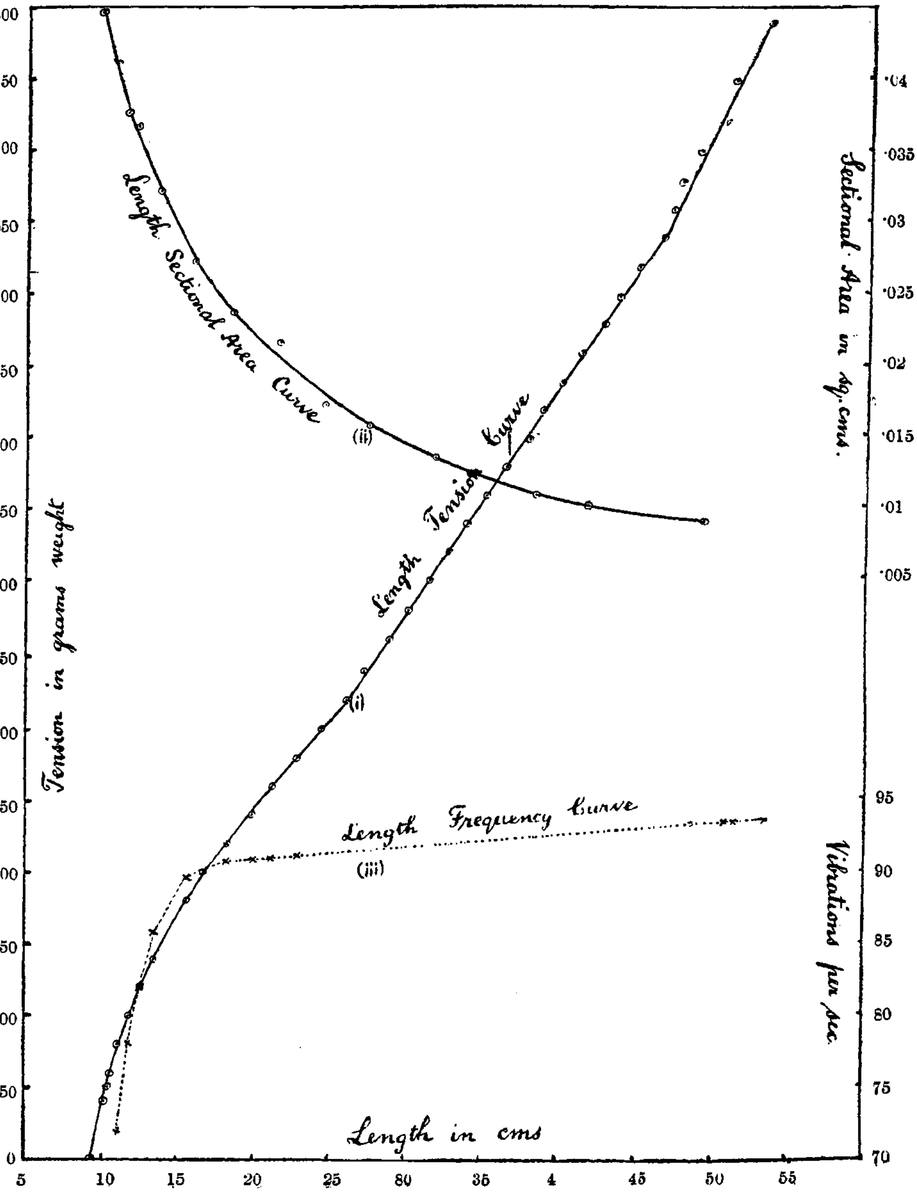
The curve connecting length with frequency shows that while the cord was doubling its natural length the pitch of its note was rising rapidly; but that further extension was practically without effect.

When the cord had doubled its natural length, its frequency of vibration was 91; and when its original length had been sextupled the frequency was not more than 94.

The discussion which follows relates to the conditions prevailing during the period in which length and tension are connected linearly.

Since the observations show that equal increments of tension (between the limits above mentioned) produce equal extensions; and at the same time the sectional area is diminishing, it appears that the value of Young's modulus must increase with increasing extension.

Experiments have been made on this point by Villari (Pogg. *Ann.* cxliii.), by Röntgen (*ibid.* clix.), and by Mallock (Proc. Roy. Soc. 1889); but beyond ascertaining that Young's modulus increases rapidly with increase of extension no definite relations appear to have been discovered.


Tension in grams weight
Sectional Area in sq. cms.
Vibrations per sec.
Length in cms
Length Sectional Area Curve
(ii)
Length Tension Curve
(i)
Length Frequency Curve
(iii)
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
·04
·035
·03
·025
·02
·015
·01
·005
95
90
85
80
75
70
5
10
15
20
25
30
35
4
45
50
55

The values of the modulus for the cord used in these experiments were therefore calculated by the use of curves (i.) and (ii.).

In each case an extension of 1 cm. was chosen, and the increase of tension required to produce this was 16 grams weight.

The mean value of the sectional areas before and after each increase of tension was used in the calculation; and for this purpose curve (iii.) was employed.

When the results are tabulated, it becomes apparent that *Young's modulus is proportional to the square of the stretched length of the cord.*

The last column in the table exhibits this relation.

TABLE showing Relation between the stretched Length of an Indiarubber Cord and the corresponding Value of Young's Modulus.

Natural dimensions of the cord 9·3 cm. × ·22 cm. × ·221 cm.
Frequency of the stretched cord = 93.

Original Length in cms.	Final Length in cms. (L).	Sectional Area in sq. cms.	Increment of Tension in grams weight.	Young's Modulus. (Y).	$\frac{Y}{L^2}$.
26	27	·0159	16	26163	35·9
28	29	·0148	16	30270	36·0
30	31	·0138	16	34782	36·2
32	33	·0128	16	40000	36·7
34	35	·0121	16	44958	36·7
36	37	·0112	16	51200	37·3
38	39	·01057	16	57508	37·8
40	41	·0101	16	63366	37·6
42	43	·0097	16	69278	37·4
44	45	·0094	16	74734	36·9
45	46	·00927	16	78431	37·0

If this relation is borne in mind, it can be shown that the frequency of a greatly stretched rubber cord will increase very slowly indeed with increase of tension.

Let T = tension.

l = natural length of the cord.

L = stretched „ „

m = mass of unit length of cord.

w = mass of cord.

ρ = density of the rubber.

s = sectional area of the cord.

Y = Young's modulus.

Then between the limits previously specified,

$$\frac{dT}{s} = Y\frac{dL}{L}.$$

But

$$\rho s L = w:$$

$$\therefore \quad \rho\frac{dT}{w} = Y\frac{dL}{L^2}.$$

Now since it has been shown that

$$Y = kL^2;$$

$$\therefore \quad \rho\frac{dT}{w} = k.dL\ ;$$

$$\therefore \quad \frac{\rho}{w}T = kL + C^*.$$

Put T = 0 when L = l,

$$\frac{\rho}{w}T = k(L-l).$$

But

$$w = Lm,$$

$$\therefore \quad \frac{\rho T}{Lm} = k(L-l);$$

and

$$\frac{1}{L^2}\cdot\frac{T}{m} = \frac{k}{\rho}\left(\frac{L-l}{L}\right).$$

Also for a vibrating string

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}};$$

$$\therefore \quad n \propto \sqrt{\frac{L-l}{L}}.$$

* This assumes ρ constant. Villari found the density of rubber which was stretched four times its natural length to be ·966 times its natural density.

The value of this expression increases very slowly with increase of L if the latter is several times l. This appears to be the explanation of the remarkable constancy in pitch of the note given by the vibration of a rubber cord under increasing tension *.

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X. *The Relative Rates of Effusion of Argon, Helium, and some other Gases. By* F. G. DONNAN, *M.A., Ph.D., Junior Fellow, Royal University of Ireland* †.

Introduction.

WHILE investigating argon and helium, Prof. Ramsay and Dr. Collie ‡ have found that these gases *diffuse* through a porous plug into a vacuum with velocities which are higher, when compared with a standard gas such as oxygen, than those calculated according to the law of the inverse square root of the density. The object of the present investigation is to ascertain the relative rates of flow of these gases through a small hole in a thin-walled partition, *i. e.* their relative rates of *effusion.*

The subject of the efflux of fluids was investigated by D. Bernouilli §. Since that time the subject has been investigated both theoretically and experimentally by numerous writers. In the present case we have only to deal with the efflux of gases. So long ago as 1804 Leslie ‖ suggested the determination of relative densities by this means, and in fact described the method which was afterwards worked out by

* Since this was written, the author's attention has been directed to a paper on the same subject by von Lang (Wied. *Ann.* vol. lxviii.). The constancy of pitch is briefly mentioned as due to an observed proportionality between the tension and the stretched length of the rubber cord. The paper deals chiefly with small discrepancies between the frequencies observed and those calculated from Taylor's formula.

† Read March 2, 1900.

‡ Proc. Roy. Soc. vol. lx. p. 206.

§ *Hydrodynamica*, 1738, Sec. 10, pr. 34, p. 124.

‖ 'Experimental Inquiry into the Nature and Propagation of Heat,' p. 534.