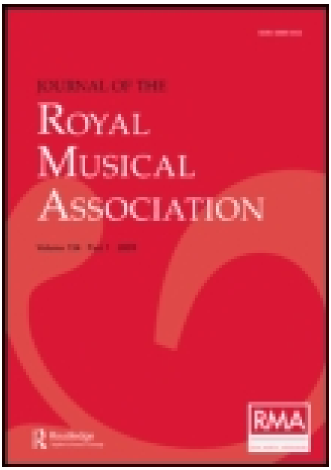

Proceedings of the Musical Association
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rrma18

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Published online: 28 Jan 2009.

DECEMBER 6, 1875.

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ON THE GRAPHIC METHOD OF REPRESENTING MUSICAL INTERVALS. WITH ILLUSTRATIONS OF THE CONSTRUCTION OF THE MUSICAL SCALE.

By WILLIAM POLE, Esq., V.-P.R.S., Mus. Doc. Oxon.

THE object of this paper is to explain a method of representing Musical Intervals, which is very useful in the higher class of musical investigations, from the easy and definite manner in which it enables relations, usually considered complex and obscure, to be presented to the mind.

If we think a little, we shall find that we have a natural tendency to compare the positions of musical notes with positions in space. It is not clear that there is any real physical or physiological connection between the two things; but it is certain that the idea of such a connection has, somehow or other, become implanted in our minds.

For example, we call a note with very rapid vibrations a *high* note, and one with slow vibrations a *low* note; clearly referring them to comparative positions in space. I do not know whether it has ever occurred to you, that these expressions, high and low, are purely arbitrary and conventional; there is no natural justification for them, but they have existed almost ever since music took a definite form, and they clearly illustrate the analogy I am speaking of.

Now further, if a rapid-vibration note is called high, and a slow-vibration one is called low, it follows that the *musical idea of distance*, or what we call the musical *interval*, between the two notes may be considered as having an analogy, in our minds, with the *interval in space* between the high position and the low one. And we can easily imagine that if the musical distance between the two notes is greater, it may be represented by a greater interval of space, and *vice versâ*.

In other words, it is possible, and consistent with ideas already existing in our minds, that representations of musical intervals should be made for the *eye*, so as to convey ideas of comparative magnitude precisely analogous to the impression which these intervals make on the *ear*.

This I call the GRAPHIC method of representing intervals; and my object now is to show how this mode of representation may be reduced to rule and system, and put in a practical shape for use.

I shall have, by-and-by, to give you an illustration of the system in the graphic representation of the musical Scale; and I may at once remind you that the idea of such a representation is embodied in the very word itself; for our term *Scale* is derived from the Latin *Scala*, which means a ladder or staircase; and this, of course, implies that the intervals of *space* between the various steps of a ladder have been considered, from very remote antiquity, as corresponding with the *musical* intervals between the various notes of the scale.

Mr. Hullah has long ago given a practical form to this idea, by using the symbol of a ladder in some of his elementary books, to represent the diatonic major scale; and there is a point in his diagram which specially bears on my present subject, namely, that he has made the intervals between the third and fourth, and between the seventh and eighth steps of his ladder, only half the length of the other degrees, thereby expressing, in a graphic mode, the distinction in magnitude between the whole tones and the semitones. This is really the germ of what I am now going to show you. All I profess to do is to establish the mode of representation on more definite principles, and give it more capability and more accuracy.

Suppose we were to attempt to lay down two distances on paper, representing two intervals of different magnitudes. On the principle of *equal temperament*, this would be very easy; for as every interval is assumed to consist of a certain number of semitones, we should only have to take a certain unit of length, say one inch, to represent a semitone, and we should have—

An octave . . .	12	inches	long.
A fifth	7	"	"
A fourth	5	"	"
A major third . . .	4	"	"
A minor third . . .	3	"	"
A whole tone . . .	2	"	"

and so on.

But now suppose we want to go more accurately into the harmonic relations, by representing the magnitudes of *true* intervals, or intervals perfectly in tune—how should we set about this? Or, indeed, since it is not at all necessary to the philosophical definition of an interval that it should be in tune, or should be of any particular magnitude whatever, let us ask the more comprehensive question, What guide or rule have we for the graphic representation of an interval generally?

The problem then is, Given any two musical sounds, by what rule shall we find a distance in space representing the interval between them?

The first thing is to define accurately, in some scientific way, the *positions of the notes*; and the simplest way of doing this is, by their vibration-numbers.

Suppose, for example, that the upper note is due to 200, and the lower one to 100 vibrations per second, the musical interval being, as we know, an octave. Then, the difference between these being 100, it would be very natural to say, 'Let us take any space represented by 100 inches, or 100 equal parts of any sort, and this shall represent the octave interval between those two sounds.'

But the simplest mode of procedure is not always the right one; and that is the case here, as can easily be shown. Let us try, on the same plan, the interval between two other notes, of 200 and 400 vibrations respectively; which, as you know, will also be an interval of an octave. Here the difference will be 200; so that if we measure by the number of vibrations, we get two different values for the same interval, in different parts of the scale.

Of course this will not do; the same interval conveys the same musical idea of distance, wherever it be taken, and therefore any rule which is to determine its representation by space, must give, under all circumstances, the same value.

There is such a rule, but it involves a hard word; as it requires the use of what mathematicians call *logarithms*. But the difficulty is more in the name than in the reality, for although the theoretical nature and construction of logarithms require somewhat high science to understand, yet fortunately the practical *use* of them (which is all we have to do with here) is extremely easy—as easy, in fact, as a child's sum in arithmetic.

There are published what are called 'tables of logarithms,' by which the logarithm for any number may be found by simple inspection; and with the aid of these the rule for representing the exact magnitude of any interval becomes very simple.

> *Find in the table, first, the logarithm corresponding to the vibration number of the upper note, and then that for the lower note. Subtract the latter logarithm from the former, and the remainder will be a number which will correctly represent the interval desired.*

To show you how easy this is, I will apply it to the two cases above named. I may say that the published tables give the logarithms in several figures—five, six, seven, or more; but it is only the left-hand ones that are of importance for our purpose. The right-hand ones are of very small significance; and if we take enough from the left hand to give the required interval in three figures, it will be sufficiently accurate.

The first case was for notes having the vibration numbers 200 and 100 respectively:

The logarithm of 200 is found by the table to be* .	2,301
The logarithm of 100 is	2,000
Hence the interval of an octave is expressed by the number	301

The second case is for the numbers 400 and 200:

Logarithm of 400	2,602
Logarithm of 200	2,301
Interval of the octave as before	301

Take another example, say from middle C, of 264 vibrations to the G above, which has 396:

Logarithm of 396	2,598
Logarithm of 264	2,422
Hence the interval of a fifth will be expressed by the number	176

This is the general rule.

But for *ordinary* intervals, we may further simplify the operation. For instead of taking the vibration numbers of the two notes, we may simply take the two numbers forming what is called the *ratio* of the interval.

As a further example of this, I will now not only calculate the value of some of the best known intervals, but I will proceed to give them actual dimensions before your eyes on this sheet of paper.

For the Octave.

Ratio 2 to 1.

Logarithm of 2	301
Logarithm of 1	000
Interval of the octave	301

For the Fifth.

Ratio 3 to 2.

Logarithm of 3	477
Logarithm of 2	301
Interval of the fifth	176

* The operator must recollect the usual rule, to put *before* the number found in the table, another number, being one less than the number of figures in the vibration number.

For the Fourth.

Ratio 4 to 3.

Logarithm of 4	602
Logarithm of 3	477
Interval of the fourth	125

For the Major Third.

Ratio 5 to 4.

Logarithm of 5	699
Logarithm of 4	602
Interval of the major third	97

For the Minor Third.

Ratio 6 to 5.

Logarithm of 6	778
Logarithm of 5	699
Interval of the minor third	79

For the Major Sixth.

Ratio 5 to 3.

Logarithm of 5	699
Logarithm of 3	477
Interval of a major sixth	222

For the Minor Sixth.

Ratio 8 to 5.

Logarithm of 8	903
Logarithm of 5	699
Interval of a minor sixth	204

And so on for any intervals whatever, no matter whether consonant or dissonant, or even whether they belong to the scale at all. If we only can identify the two sounds, we get by this simple process an accurate representation of the interval between them, in a way far exceeding any other method in intelligibility and practical clearness.

You may now see for yourselves how admirably these results correspond with the musician's ordinary practical notions of things.

For example, add some of the intervals together:

A fifth	176
Added to a fourth	125
Make an octave	301
A major sixth	222
Added to a minor third . . .	79
Make an octave	301
A minor sixth	204
Added to a major third . . .	97
Make an octave	301
A major third	97
Added to a minor third . . .	79
Make a fifth	176
A fourth	125
Added to a major third . . .	97
Make a major sixth	222

And so on; these determinations representing most accurately the idea of the magnitudes of the various intervals, as we are accustomed to know them by their relations in practical harmony.

(*The above remarks were illustrated by diagrams.*)

Illustration of the Construction of the Musical Scale.

I propose now to give you a more extended practical example of the graphic method of representing intervals, by applying it to the construction of the modern musical scale, which I shall effect before your eyes.

I have already alluded to Mr. Hullah's ladder. I am going to make a similar diagram, only I shall endeavour to do it more accurately, by giving to each of the steps the exact and proper length it ought to have. I shall draw it also to a pretty large size, so that the minute shades of difference may be made very distinct to your apprehension.

Now, settling the exact notes of the scale is not quite so easy a matter as is generally supposed. I avoid at present all reference to the scale of keyed instruments; we may take it for granted that they, however well tuned, only give an approximation to correctness. If an accomplished vocalist or a violinist were asked to sound the scale, we should probably get it nearer; but we want now positive definition and absolute correctness.

The first thing, therefore, we have to do is to settle some *principle* on which we must proceed; and after a good deal of consideration, the only one that seems to me to be trustworthy for our modern music is,

That every note of the scale must have some definite HARMONIC RELATION *to some* OTHER *note.*

This would seem obvious enough; but, as I shall show you hereafter, it leads to some points of controversy, and has been dissented from by good practical authorities. I must, however, adopt it here, as I do not see on what other principle we could construct a scale of true intonation.

The next question is, *What* harmonic relations shall we make use of in forming the scale? Omitting the octave, which we may consider as a duplicate of the key-note itself, we only require two, namely, the two which are given by nature in almost all musical sounds; which were taken by Rameau, the founder of the modern theory of harmony, as his basis; and which have formed ever since the first elements of harmonic science. These are the *fifth*, and the *major third*: forming the triad, or common chord. With these we can do all that is necessary to form, not only the diatonic scale, but all the accidentals adjoining.

Thus, taking C as the key-note,

A fifth	above	C	gives	. .	G
A major third	„	C	„	. .	E
A fifth	„	G	„	. .	D
A major third	„	G	„	. .	B
A fifth	below	C	„	. .	F
A major third	above	F	„	. .	A

(*The whole of these determinations were illustrated by diagrams.* *)

We have thus got the whole of the *Diatonic Major Scale*; and we shall find, for the most part, the harmonic relations between its several notes well preserved.

But in some cases this is not so. For example, the interval,

D — A

is found not to be a true fifth; the A being one fifth of a semitone too flat.

If we were to alter the A by making it a true fifth to D, we

* Diagrams of a similar nature, contributed by the author of this paper, will be found in an appendix to Sir F. A. G. Ouseley's *Treatise on Harmony*. Although our modern musical scale may be taken to be constructed on harmonic principles, as here laid down, yet it must not be forgotten that a scale very similar was in use for centuries before what we now call *harmony* had any practical existence. The origin of this ancient melodic series of notes involves some very interesting but abstruse speculations, which it would be impossible to introduce here.

should put it out of tune for the subdominant chord F A C. If, on the other hand, we altered the D, we should spoil the chord of the dominant, G B D; and as both these chords are of such importance to the harmonies belonging to the key, it is better to retain them pure, leaving the error in the less important fifth D A.

We shall also find that from D to F is not a true minor third. To remedy this we must either sharpen the F, which will spoil the subdominant fifth F·C, or we must flatten the D, which will spoil the dominant fifth G D. But both these are so important, that they must remain true, and hence we must put up with a bad minor triad on the second of the scale: bad in two ways, both its third and fifth being wrong.

Before we go further, a remark or two on this imperfection present themselves.

In the first place it gives good evidence, to my mind at least, that the diatonic scale is a conventional and artificial series of notes, and not, as many people suppose, one dictated by any natural, physical, or physiological laws. If it had been so, it might have been expected to be perfect; whereas it is essentially imperfect, by its very nature and construction. We are so much accustomed to it, that we are apt to think that it is the true natural foundation of musical melody. But there are many facts which oppose this idea, and this is one of them. It is true that it lends itself easily to certain naturally harmonious and pleasant combinations; and this is, I believe, all that can be said in favour of such a view.

Secondly, in these out-of-tune relations between D, F, and A, we come upon the first and simplest of the difficulties in the way of getting what is called *just* or *true intonation* on keyed instruments. It is clear that no instrument with seven notes in the diatonic scale can be always in tune for that scale. We want other notes in addition, namely, alternatives for D, F, and A. We shall find other difficulties develop themselves as we proceed.

I will now go on to put in some of the notes called *accidentals*—sharps and flats.

Although these do not belong to the key in which we have been working, yet, as we must determine on some way of defining their places, the rule before adopted will still hold good; they must have harmonic relations with some of the notes in the diatonic scale. And, as we shall see, the same simple relations will suffice; in fact, with the major third, we can do all that is necessary.

For the Sharps.

A major third above	D	gives	F♯
,,	,,	A	,,	C♯
,,	,,	E	,,	G♯
,,	,,	B	,,	D♯
,,	,,	F♯	,,	A♯

For the Flats.

A major third below		D	gives	B♭
„	„	G	„	E♭
„	„	C	„	A♭
„	„	F	„	D♭
„	„	B♭	„	G♭

and we might go further in each case, if desired.

We have, however, in the above list got all the most common chromatic notes; but I am bound here, in candour and fairness, to mention an objection that has been raised to the accuracy of these determinations.

The diagrams will show that, comparing the two accidentals C♯ and D♭, or G♯ and A♭, the *flat* note is much the *higher* of the two. Now, if there is any violin player present, he will at once say that this is contrary to what he has been taught, and what he practises. It is, I believe, one of the positive instructions to violin students that the C♯ must be the nearest to D, and the D♭ nearest to C.

Berlioz, no mean authority, considered the discrepancy so important and so positive, as to throw doubt on the whole theoretical system of harmonic relations; and an excellent little book on the elementary principles of violin playing, lately published under the sanction of no less a personage than Herr Joachim, embodies the instruction as to the relative position of the two notes in unequivocal terms.

I have felt this to be a very serious difficulty; for it would be out of the question to ignore the opinions of well-educated instrumentalists on such a question, while at the same time the principles on which these harmonic relations are constructed are so simple, and have been so clearly proved, both by theoretical and practical considerations, by every investigator, from Pythagoras to Helmholtz, that it would be equally out of the question to controvert *them*. I have never seen any attempt to explain or to reconcile this curious discrepancy. I have some ideas on the subject myself, but it is too complicated to enter into now: possibly I may state them at some future meeting of this Society.

In the meantime, as in duty bound, I mention the difficulty, and commend it to the attention of thoughtful musicians.

Recurring now to our scale, you will begin to see further the complexities and difficulties of true intonation. We have already provided for eleven keys, and have got seventeen different notes in an octave; but to get the accidentals required for modern music we ought to have gone still further. Moreover, there is another kind of difficulty. Even the keys that we have nominally provided for are not in tune; for if we test the series of notes just found by the diatonic scales of different keys, we shall find some such result as follows:

In the key of	A	the intervals of the	2nd	will be out of tune.
„	D	„	2nd, 5th, 7th	„
„	A	„	4th, 6th	„
„	B	„	2nd	„
„	F	„	4th, 6th	„
„	B♭	„	2nd, 5th, 7th	„
„	E♭	„	2nd	„
„	D♭	„	4th, 6th	„

It is evident, therefore, that to get perfectly just intonation in all these keys, we must, in addition to the seventeen notes actually drawn, have many of them in duplicate or triplicate; and hence it is that all the science and industry of Col. Thompson, Mr. Ellis, Mr. Bosanquet and others, have been exerted to establish various *systems* of *intonation*, with the object of producing music perfectly or approximately in tune.

Now, I need hardly tell you, that in order to get over all this fearful complication, some ingenious person, a long time ago, was clever enough to see that by a process of *compromising*, it would be possible to simplify enormously the construction of the scale, particularly in its chromatic parts. He saw, in the first place, that the distance between E and F, and between B and C, was *nearly* half that between C and D, or G and A. He also saw that C♯ and D♭, G♯ and A♭, &c., were not very different from each other; and putting all these things together, he saw that if he divided the octave into twelve equal parts, he would produce a set of notes not much differing from the true ones, and with the wonderful simplification of being applicable to all keys. Hence arose the Modern Pianoforte Clavier, and the system of *equal temperament*, so well known.

I will now construct, graphically, an equal-tempered scale, and compare it with the one already before you. And this is the easiest thing possible: I have only to divide the octave into twelve equal spaces, and the thing is done. Each of these spaces represents a semitone, and two of them make a whole tone.

(*This was done on a diagram.*)

It is now very easy to compare the equal-tempered with the just scale, in order to see the nature and magnitude of the errors resulting from the compromise. The most important are as follows:—

Equal-tempered notes.				
C		Right.		
B	Major 7th	Too sharp	by $\frac{1}{8}$	of a semitone.
B♭	Minor 7th	Too flat	„ $\frac{1}{4}$	„
A	Major 6th	Too sharp	„ $\frac{1}{6}$	„
A♭	Minor 6th	Too flat	„ $\frac{1}{7}$	„
G	Fifth	Too flat	„ $\frac{1}{50}$	„
F	Fourth	Too sharp	„ $\frac{1}{50}$	„
E	Major 3rd	Too sharp	„ $\frac{1}{7}$	„
E♭	Minor 3rd	Too flat	„ $\frac{1}{6}$	„
D	Major 2nd	Too flat	„ $\frac{1}{25}$	„
C		Right.		

I have no wish, in this paper, to lead to a discussion on the vexed question of temperament; I will merely state, in a few sentences, what my own general views on the subject are.

In the first place, I do not agree with the thorough-going theorists who delight in reviling and despising the unfortunate equally tempered scale. On the contrary, I hold that it has been one of the most happy and ingenious simplifications ever known in the history of music; that this simplification has been the means of advancing the art to an incalculable extent; and that the modern enharmonic system, founded upon it, is so thoroughly incorporated into modern music, that it is difficult to imagine how it could be now abandoned. And then as to performance, one fears to think of the complication that must be entailed on keyed instruments if equal temperament were forbidden. The pianoforte, to which we are indebted for perhaps nine-tenths of the music in the world, could hardly be said to exist if it were attempted to put it in mathematical tune.

But having said this, I am equally at variance with that other extreme party who would force upon us the equal temperament in cases where perfect intonation can be obtained just as easily, namely, as in stringed instruments, or pure vocal music.

I hold that the too-sharp third, whatever may be said for its great utility, is still harsh and disagreeable, and ought never to be tolerated in sustained tones, if the natural and true harmony can be got. It is the possibility of getting this which gives such an inexpressible charm to stringed and vocal harmony when unaccompanied by the intractable keyed instruments; and in the true interests of sweet music, this kind of perfection ought to be encouraged by every means in our power.

The equal temperament is to a large extent a necessary evil; but *no* evil ought to be tolerated when it is *unnecessary*.

DISCUSSION.

Mr. de Pontigny said that he was delighted to hear a scientific man admit that the equal temperament scale was possible. They had had much discussion on the subject.

The Chairman said that there had been frequent allusions to him in the paper, and it was rather unfortunate that the attitude which he, amongst others, had taken up with respect to this question should be, as he conceived, entirely misunderstood. He had never said that a tempered scale was impossible, or that he did not use it himself. He had always said and maintained that, for many practical purposes, the tempered scale must always be used; but why should they shut their eyes to all other possible forms? Why should they not develop the resources of their art?

In optics many minute questions were debated by the intellects of the country, and debated with such a power of thought that one would think that they were questions of national existence, while very often they had not even one practical application. Very often questions exactly of the same kind arose in music, but they were capable of practical application; and certain persons had tried to investigate them, and to put the difficulties before people in a simple manner, and show that these questions were not devoid of practical application. But when this was done it was immediately said that they wished to overturn the whole system of music as it at present existed. Nothing could be farther from the thoughts of himself and others who took up such questions. What they wanted was; that the material of music should be studied, and when that point was attained it was always followed, in the long run, by some practical advantage. Seeing that there were at present most interesting problems —problems of quite as much interest as those which existed in optics—why should not musicians study them? Having said so much—which, perhaps, was a little too much—he would now refer to the subject which had been dealt with in the paper. There were just one or two little points that he should like to mention. Three or four years ago, in giving a small course of lectures at Oxford to some musicians, he found it rather a good opportunity for ascertaining what was the best method for graphical representation—exactly the very point which Dr. Pole had been bringing before the Association; and he hit upon a method which he found extremely successful. He sketched an ordinary pianoforte key-board, in very light lines, upon a very large sheet of paper, the scale of the drawing being immaterial; and he shaded in the black keys very lightly. He then took the middle point of each key to represent the pitch of the corresponding note, and whatever he wanted to lay down he laid down from that as the scale. For instance, for harmonics he divided the key-board into portions corresponding with the number of octaves, and introduced lines corresponding to the harmonics. The result was that he had the whole of the harmonic scale mapped out, with every note in its exact place upon the pianoforte key-board—a representation which every musician knew intimately the meaning of, and which could not possibly be misunderstood. He never had any difficulty with the harmonic scale after that. Another point which struck him was as to the way in which the sharps were derived. Of course, historically speaking, they were in the first instance derived, he supposed, from what was called the old Pythagorean system, which was always that the sharps were in the ascending system of fifths, and the flats in the descending. Then gradually the fifths were flattened, and in the sequence they always retained the same value; but it was only when the fifths became flattened, as they were in the mean-tone system, that it was possible to use the C sharp as the third to A. They could not do that in the old Pythagorean system; but it was merely a question of general usage, and he thought that if they looked to the books which were said to be authoritative on these points—though, for himself, he did not profess to know anything of authority—they would find that the only meaning that was attached authoritatively by modern musicians to the symbols of sharps and flats was that they represented proceeding upwards or downwards in a consecutive series of fifths. In the papers that he had read about this subject he had invariably employed equal temperament semitones as the unit of interval. Of course that arose out of the employment of the graphical method which he had just been describing. When he found that it was easy to show the intervals by laying them down upon the scale of pianoforte keys, it was the simplest thing in the world to find rules for expressing intervals in numbers having relation to that scale. That is to say, he always regarded the perfect fifth as seven equal-temperament semitones and a fraction. Thus:—

$$7{\cdot}01955 \text{ or } 7\,\tfrac{1}{51{\cdot}151}$$

He had got so accustomed to this that it seemed probably more simple to him than to other people; but he thought that, while the numbers of the logarithmic scale conveyed in themselves no idea to the practical musician, unquestionably the numbers of a scale such as he employed did convey the idea in a way most easily and clearly understood by a practical musician.

Dr. Pole said that he had really nothing to say in reply, because there was nothing in which he disagreed from Mr. Bosanquet. It must not be supposed that in any remarks he had made in defence of the equal temperament he depreciated what Mr. Bosanquet and others had done. As to the relation of the true notes to each other, and the systems and modes of temperament, all that Mr. Bosanquet had written was exceedingly good, and deserving of study. He (Dr. Pole) merely wished to put on record his idea—which Mr. Bosanquet himself admitted—that in a practical point of view equal temperament had almost incalculable advantages; and that to it they were indebted, very much more than they considered, for the extension of the art of music. As to the use of the pianoforte keys, taking the equal-tempered semitone as the unit was a very good simplification; but he was not quite sure that a quantity like 7·01955, used to represent the perfect fifth, was a simpler thing than the logarithm scale. However, that was not very important, for if the thing was to be done graphically it must be founded upon logarithms in some way or other; and therefore there was really nothing in dispute. As to the remark made about the sharps, when, with some hesitation, he drew his diagram, he said that he did not exactly know what C sharp was, unless it was the major third to A; and therefore, in drawing C sharp, he did not know where else to put it. He was aware at the time that it was *open to objection*; but still, what was one to do? Let them ask a musician what C sharp was, and if he could tell them anything else than that it was the third to A, he should be rather surprised, because that was the musician's idea of it. If it was not that, what was it? It might be one of a series of fifths derived in a roundabout way, but the musician never thought of it so. He thought of A flat as a third below C. It was exactly in these points that doubt arose. He had taken the notes in the diagram in the simplest way he could; but that did not affect the principles of the Paper, for if anybody had any other way of defining C sharp he could do it just as easily on his (Dr. Pole's) method; and he claimed for this plan that it was a mode of representing to the eye the relations of the intervals in a way corresponding to the impressions they made upon the ear; and therefore it was useful, as it gave them, as they had found to-night, the opportunity of reasoning about the things in a better way than they could do by abstract words.