ON THE DIFFERENTIAL TELEPHONE.[1]

By William Duane and Charles A. Lory.

1. The authors have tested the merits of a "differential telephone" as a means of measuring a self-inductance, and have devised an exceedingly accurate method of measurement. The differential principle has been used before in connection with a telephone,[2] but the accuracy claimed for it in the measurement of a self-inductance was only about one per cent. With the instrument described below an accuracy greater than one fiftieth of one per cent. has been attained.

2. A differential telephone receiver is one on the bobin (or bobins) of which two coils AA' and BB' (Fig. 1) are wound side by side. By suitable means these coils are adjusted so as to have equal self-inductances and equal resistances, and so that the magnetizing effect of a current flowing through one coil from A to A' is exactly annulled by that of an equal current flowing through the other coil from B to B'. A variable self-inductance standard S is joined in series with coil AA', and the coil of unknown self-inductance X in series with coil BB'. A non-inductive variable resistance R is inserted in series with S or X according as the

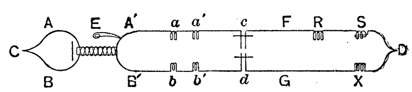

Fig. 1.

resistance of X is greater or less than that of S. The two entire circuits are then joined in parallel and an alternating electromotive force is applied to the branch points C and D. By varying R and S values can be found easily such that no sound is heard in the receiver. When this is the case the two parallel currents must be

[1] Presented at the St. Louis meeting of the American Association for the Advancement of Science, December, 1903.

[2] H. Ho, Electrical World and Engineer, May, 1903.

equal to, and in phase with each other, and therefore the self-inductance of X must equal that of S.

3. Before the instrument is ready for use, three adjustments are necessary, but these may be made once for all, and do not have to be repeated before each measurement.

First, the magnetic effects of the two receiver coils must be equalized. This is the most difficult adjustment, as it is impossible to wind the two coils exactly alike. The equality of the magnetic effects can be tested by joining the ends A' and B together, and sending an alternating current through the two coils in series in the direction $AA'BB'$. The same current then passes through each coil in the proper direction, and the magnetic effects are equal and opposed to each other, if no sound is heard in the receiver. In making the receiver it is best to wind the two coils side by each and as close together as possible, and each layer of turns should be evenly distributed. If in the first layer the turns belonging to coil AA' are nearer the diaphragm than those belonging to coil BB', in the second layer the positions of the turns belonging to the respective coils should be reversed, and so on alternately in the succeeding layers. Even with this careful winding the magnetic effects are not exactly equal to each other; in our instrument the difference between them amounted to the effect of a fraction of a single turn. A number of schemes for balancing the residual effect were tried, but the only one that completely extinguished the sound in the receiver was the insertion of a small coil E of about ten turns of wire in series with one of the receiver coils. E was mounted on the frame of the receiver in such a way that it could be shifted about with its plane always parallel to the common axis of the coils AA' and BB'. A position could be found for E that completely extinguished the sound, even though an alternating electromotive force of twenty volts was applied to the coils.

The second and third adjustments are the equalizations of the resistances and self-inductances of the coils. The self-inductances can be equalized by placing in series with each receiver coil a pair of coils (aa' and bb'), and by altering the distances between the coils in each pair. The resistances can be equalized by means of sliding contacts moving along pairs of parallel wires (c and d). The test for the equality of the resistances and of the self-inductances is accomplished by joining the circuits from A to F and from B to G in parallel, applying an alternating electromotive force to the branch points and varying the adjustments until no sound is heard in the receiver. After these adjustments have been made the instrument is ready for measurements.

4. In comparing the self-inductance of a coil X with that of a standard S two balances must be made. The resistance of the parallel circuits must be made equal to each other by varying R, and the self-inductances equal to each other by varying S. One of these balances does not depend upon the other (as is the case, for instance, in Maxwell's method of comparing a self-inductance with a capacity), and hence the double balance is much easier than it otherwise would be.

If the divisions of the resistance box R are not small enough, the final balance of the resistances may be made by means of the sliding contacts c and d.

Absolute silence in the telephone receiver can be obtained easily, if there is no iron in the self-inductance. With an iron core a very sharp minimum can be obtained, but not in general total silence. If, for instance, the coil X is a Morse telegraph sounder, several tones of different pitches can be heard, and values of S and R can be found that extinguish the most prominent tone.

5. The accuracy of the measurement may be estimated by means of the following calculation. Suppose that the resistance of the two parallel circuits are equal to each other and each equal to r, and further that the reactances of the two circuits differ from each other by the small amount dx, x being the reactance of one circuit and $x + dx$ that of the other. The complex expressions for the impedances of the two circuits are $(j = \sqrt{-1})$.

$$Z_1 = r + jx, \qquad \text{and} \qquad Z_2 = r + j(x + dx).$$

If e is the simple alternating electromotive force applied to the branch points C and D, and I_1 and I_2 are the currents in the two branches respectively,

$$I_1 = \frac{e}{r + jx}, \qquad \text{and} \qquad I_2 = \frac{e}{r + j(x + dx)}.$$

The sound heard in the receiver depends upon the difference between the two currents, and, if dI is this difference,

$$dI = I_1 - I_2 = e\left[\frac{1}{r + jx} - \frac{1}{r + j(x + dx)}\right].$$

Reducing to a common denominator, neglecting small quantities of higher orders than dx and eliminating the imaginary unit j, we find that the absolute magnitude of dI is

$$dI = \frac{edx}{r^2 + x^2}.$$

If dI is the smallest current that can be detected, when it flows through one of the receiver's coils, dx cannot be greater than

$$dx = (r^2 + x^2)\frac{dI}{e}.$$

To estimate the numerical value of the accuracy assume that the resistances r and the reactance x are each 50 ohms, that e is one volt and that a current of one millionth of an ampere can be detected by the telephone. These values are not unusual. Then

$$dx = \tfrac{1}{200} \text{ ohm},$$

and the percentage error is

$$100\frac{dx}{x} = \tfrac{1}{100} \text{ per cent.}$$

The reactance x may be taken to be that of the standard S alone, for the reactance of the rest of the branch circuit is very small. The reactance is due to self-inductance, and therefore the percentage error in the measurement of the self-inductance is also $\frac{1}{100}$ per cent.

As a matter of fact the accuracy does not fall far below the theoretical value. It is not difficult to measure a self-inductance to within one twentieth of one per cent., and with care the accuracy can be considerably increased.

6. Electrolytic resistances also may be measured by means of the differential telephone. For this purpose the coil X is replaced by a vessel with suitable electrodes and containing the electrolyte, and the standard S is removed.

7. The advantages of using a differential telephone as described above for measuring a self-inductance are: (*a*) that the apparatus is portable and does not get out of order easily; (*b*) that great accuracy can be attained and the manipulation is not difficult; (*c*) that the heating of the circuits by the currents does not alter the self-inductance balance, and therefore that large electromotive forces may be used; and (*d*) that only one standard is required; it is not necessary to know the values of any resistances or the lengths of a bridge wire. The disadvantage is that the self-inductance of the standard must equal that of the coil to be measured. If, however, the self-inductance to be measured is smaller than the smallest inductance obtainable in the standard, a fixed standard may be placed in series with the unknown coil, and the sum of their self-inductances measured; and if the self-inductance to be measured is greater than the greatest inductance obtainable in the standard, a fixed standard may be placed in series with the variable standard. Thus the range of measurement may be increased indefinitely beyond the limits of the variable standard.

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