No. 1628. "On the Mapping of a District with reference to a Central Meridian." By Francis Palmer Washington, Captain, R.E.

In the maps of the Ordnance Survey, the sheet-lines are by construction parallel and perpendicular to the particular meridian with reference to which the area represented is mapped.[1] The Author believes it to be generally conceived that the north-and-south sheet-lines lie north and south; and although approximately the case (and in those maps close to the meridian of reference it actually is so), yet it is evident that the north-and-south sheet-lines farther distant from the meridian of reference cannot coincide with their true meridians, owing to the polar convergence of all meridians. In settled and cultivated countries, the boundaries of counties or districts afford material assistance in determining the lateral range of the sheet-lines parallel to the meridian chosen, but in an unsettled or unexplored country no such aids in the first instance exist; and it may not be uninteresting to investigate the laws of the divergence of the sheet-line from its corresponding meridian, a knowledge of which will be of aid in the selection, for different portions of the globe, of the areas most suitable to be referred to one meridian.

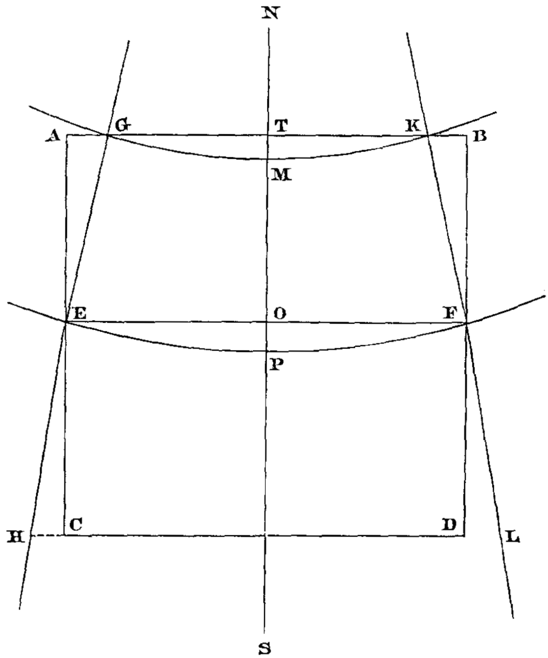
Let the line N O S represent the orthographic projection of the proposed central meridian on a plane tangential to the globe at the point O, and at right angles to the diameter passing through O; and let A B, B D, D C, C A, represent the containing sheet-lines of the area it is proposed to map with reference to N O S; A B, C D being perpendicular to, and B D, A C parallel to, N O S.

Through O draw E O F perpendicular to N O S and cutting A C, B D in E and F; and let H E G, L F K represent the projections of the meridians passing through E and F.

[1] In the Ordnance maps on the larger scales each county or group of counties has its own meridian, with reference to which the maps of the district are drawn, independently of the counties or groups of counties adjoining; *e.g.*, Chester, Kent, and Cornwall have each their own meridian, whilst one meridian serves for the group comprising Lincoln, Huntingdon, Cambridge, Bedford, Hertford, and Middlesex.

Let G M K, E P F represent the projections of the parallels passing through G, K and E, F.

Then A G represents the divergence at the point A of the straight line E A from the meridian passing through E; and if C H be equal to or less than A G, then the whole area A B D C may be mapped with reference to the meridian N O S, satisfying the condition that the divergence of A C from the meridian passing through E shall nowhere be greater than A G.



Now the arc G M K measuring the same number of degrees of longitude as the arc E P F, and the straight lines G T K, E O F representing the chords of these arcs, and being equal to the chords themselves; if R represent the radius of parallel of E P F, and r that of G M K, and if n represent the degrees of longitude measured by the arcs E P F, G M K, then $EO = R \sin \frac{n}{2}$ (1), $GT = r \sin \frac{n}{2}$ (2).

Now A G, or the divergence, $= EO - GT$; hence, from (1) and (2), $AG = (R - r) \sin \frac{n}{2}$ (3).

The equation for the radius of parallel is $R = b \cos L (1 + e + e$

$\sin^2 L)$; where b = minor semi-axis of the globe, L = latitude corresponding to radius R, e = ellipticity of the globe.[1]

Now if l = latitude corresponding to radius r, and A G be denoted by Δ, by substitution and re-arrangement (3) becomes

$$\Delta = b \sin \frac{n}{2} \left\{ \cos L \left(1 + e + e \sin^2 L\right) - \cos l \left(1 + e + e \sin^2 l\right) \right\} \quad (4).$$

By means of this equation, the position of the meridian of reference being given for a district to be mapped, the divergence of any sheet-line from its corresponding meridian can be determined for any latitude; or conversely, the limit of divergence being fixed, the area of the district to be referred to one meridian can be determined.

[1] The value of e varies according to different authors, from $\frac{1}{298 \cdot 15}$ to $\frac{1}{299 \cdot 33}$, which latter is the value assigned by Professor Airy; but for purposes of calculation in equation (4) it will be found most convenient to take it as $\frac{1}{305}$, this slight difference being unimportant.