A Modified Theory of the Crookes Radiometer

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XXII. *A Modified Theory of the Crookes Radiometer.* By GILBERT D. WEST, M.SC. (LOND.).

A striking feature of the physical literature of the period 1874-1881 is the extraordinary amount of interest that was raised by Crookes' researches "On the Repulsion Resulting from Radiation." In addition to Crookes himself, Fitzgerald, Johnstone-Stoney, Stokes, Schuster, Pringsheim, Osborne-Reynolds, and Maxwell, all gave the phenomena their attention. It was the work of Osborne-Reynolds and Maxwell, however, that was held to settle the numerous discussions that had arisen. These physicists showed, in two long and highly mathematical Papers, published in the Phil. Trans. of 1879, that the phenomena under notice were explicable in terms of the then somewhat new kinetic theory of gases. As a result of their Papers, the interest in the theory of the radiometer—great though it had previously been—subsided, and but for a criticism of Reynolds' work from Fitzgerald in 1881,* there is little worth recording for the next sixteen years. It would seem that this change occurred, largely because the majority of physicists felt that the matter had been placed beyond their grasp, rather than because some well-understood explanation had been given which rendered further research superfluous. According to Fitzgerald, Reynolds had rendered "a difficult subject tenfold as elaborate as was necessary," but however this may be, it is certain that an easily intelligible explanation of radiometric phenomena was wanted. Attempts were made, it is true, to give the substance of the accepted theories in general language, but, although many of the text book "explanations" are still based on these, it cannot be said that they met with much success.

In 1896, however, Sutherland turned his attention to the subject, and his Paper† had the advantage that, unlike those of Reynolds and Maxwell, it could be read with profit by the physicist of ordinary mathematical attainments. Yet appearing as it did at a time when the interest in the theory of the radiometer had practically ceased, it was neglected, and has since become almost entirely forgotten.

It was shown by the author,‡ however, that Sutherland's theory of the closely related phenomena of thermal transpira-

* Phil. Mag., 11, p. 103, 1881.
† Phil. Mag., 42, p. 373 and p. 476, 1896.
‡ Proc. Phys. Soc., Vol. XXXI., p. 278, 1919.

tion was so far correct, that it could be made to form an adequate basis for the careful experimental work of the Danish physicist, Knudsen, performed thirteen years later. The author was thus tempted to place considerable confidence in Sutherland's theory of the radiometer. It must be observed, however, that, at the time when Sutherland wrote his paper, little was known of the thermal surface conditions in a rarefied gas, and thus it is not surprising to find that Sutherland's theory —like many another imperfect theory in other branches of physics—will explain some of, but not all the experimental facts. It is the object of the present Paper to outline a theory of a general descriptive character which, while being to a large extent based on Sutherland's work, makes use of knowledge gained since that time.

Sutherland, like Reynolds, bases his theory of the radiometer on the closely related phenomena of thermal transpiration. He first considers the case of a tube along which a temperature gradient is maintained. If the tube connects two infinite spaces, he shows that an even flow of gas takes place over the whole cross section of the tube from the hot to the cold side. The walls of the tube in thus constraining the gas to take their temperature, exert a tangential traction upon it, whilst they themselves experience a reaction in the opposite direction. A nett reaction is only experienced, however, near the entrance of the tube, where the velocity of the gas rises from zero to the uniform value eventually attained. At all other parts of the tube the friction of the moving gas against the walls of the tube exactly balances the traction the walls exert on the gas ; it is to the "unequilibrated traction," however, that Sutherland attaches most importance.

If the spaces the tube connects are finite, the uniform flow of gas produces a pressure in the hot vessel, which in turn produces a flow of the Poiseuille type in the reverse direction to the original flow. The result of the superposition of these two flows is to give a gas current from the cold to the hot side along the walls of the tube, together with a current from the hot to the cold side along the axis, whilst between the two currents there is a surface of zero velocity.

With reduction of gas pressure, the Poiseuille counter flow becomes less and less important, until at the highest rarefactions it is negligible, and the hot regions are then enabled to maintain undiminished their higher pressures.

Sutherland points out that both the "unequilibrated traction" and the thermal transpiration pressure operate in causing radiometer motion, and to make his meaning clear he takes the case of a piston that fits loosely into a cylinder closed at both ends. He imagines the temperature of one compartment to be higher than that of the other, and that a temperature gradient exists in the material of the piston. As a consequence of the operation of thermal transpiration, the pressure on the hot side will become greater than that on the cold side, and a force will be exerted on the piston, both as a result of this excess pressure in the hot compartment, and as a result of the unequilibrated traction on the sides of the piston. By increasing the clearance between the piston and the cylinder, the thermal transpiration pressure can be made to bear an ever-decreasing ratio to the total thrust on the piston. If the piston be free to move, it constitutes, according to Sutherland, an exaggerated example of radiometer motion, and he formulates his theory on these lines.

It must be admitted, however, that to pass from the case of the piston to that of the vane of a Crookes radiometer, is a big step. Nevertheless, Sutherland maintains that such a step is legitimate and proceeds, with considerable success, to analyse the experimental data obtained with a type of torsion radiometer. Except in so far as it was better suited to give quantitative measurements, this type did not differ materially from the familiar type in which flat circular vanes, blackened on one side and silvered on the other, are mounted at the ends of a pivoted arm. "When the black face is irradiated," he says, "there is a fall of temperature . . . through the thickness of the vane, and thus the thickness of the vane becomes a surface capable, along with the surface of the bulb opposite it, of starting thermal transpiration from the cold edge to the hot, with elevation of pressure in front of the hot face . . . and depression of that behind the cold face."

Such, briefly, is an outline of Sutherland's theory, but it is clear, as indeed he himself states, that it is based on the idea that the effect of the introduction of a heated body into a gas is "to make the layer of gas in contact with the solid take the temperature of the solid at every point of the surface." It has since been realised, of course that this is not so, and that in any calculation made in regard to rarefied gases, due allowance must be made for the temperature discontinuity at the surface of the solid.

To make what is meant more clear, consider the case of two large parallel plates—one hot and the other cold. We may regard the molecules that strike the hot surface, both as coming from a distance of the order of the mean free path, and as possessing the temperature of this region. If, after reflection at the surface, these molecules merely acquire its temperature, it is clear that the mean temperature of the surface layer of gas is necessarily lower than that of the surface itself. As a matter of fact, however, it has been shown* that, on the average, impinging molecules do not even acquire the temperature of the surface they strike, and hence the discontinuity already noted becomes enhanced.

Thus in the light of modern research, one of Sutherland's assumptions is seen to be unjustifiable, and the theory built upon it must therefore be imperfect. It is possible, however, to modify the theory.

Consider once again the case of the hot and cold plates, but now with reference to the change that takes place in the isothermals, as the pressure is lowered. It is clear that, with increasing mean free path, the extreme isothermals will disappear, and that eventually, when the mean free path becomes large compared to the distance apart of the plates, the gas between will reach a uniform mean temperature. Reduction of gas pressure will thus be accompanied by a gradual diminution, and eventual elimination, of the temperature gradient between the plates.

In the particular case considered, it is easy to calculate the positions of the isotherms with changing gas pressure, but in the case of a radiometer vane this is much more difficult. However the present purpose is served quite well if a general indication is given of the changes that take place.

To make matters simple, consider first of all the case of a disc with the hot surface A 10° C. above the cold surface B—the walls of the containing vessel being supposed distant, and at the mean of the temperatures of the hot and cold surfaces. In Fig. 1 the isotherms in the gas are drawn for the case where the gas pressure is very high, whilst in Fig. 2 a

* Soddy & Berry, Proc. Royal Soc., A, 84, p. 576, 1911; Knudsen, "Ann. d. Phys.," 34, 4, p. 593, 1911; Smoluchowski, "Phil. Mag.," XXI., p. 11, 1911; Langmur, "Phys. Rev.," Vol. II., No. 5, p. 329, 1913.

rough idea is given of their appearance at a lower pressure. It will be seen that the extreme isothermals have disappeared, and that, so far as the gas is concerned, neither surface of the disc coincides with an isothermal surface. On the contrary, a temperature gradient extends from the circumference towards the centre.

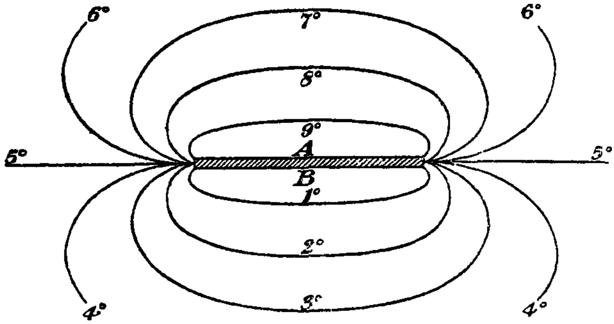

FIG. 1.—ISOTHERMS AROUND RADIOMETER VANE—PRESSURE HIGH.

Such a temperature gradient in the surface layer of gas was encountered in the experiments described in a previous paper by the author on the forces acting on heated metal foil surfaces in rarefied gases,* and it was there shown that satis-

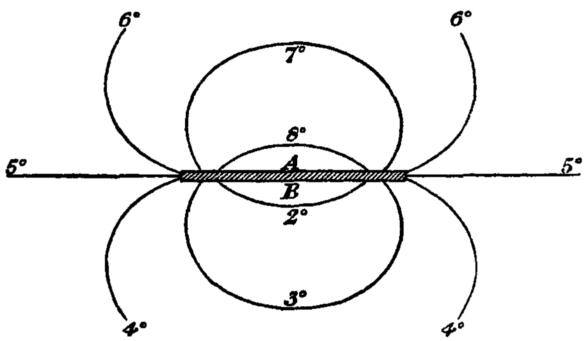

FIG. 2.—ISOTHERMS AROUND RADIOMETER VANE—PRESSURE LOW.

factory explanations could be based on the assumption that this temperature gradient gave rise to gas currents somewhat similar to those arising in a tube in the material of which a temperature gradient was maintained in the direction of the length.

* "Proc." Phys. Soc., Vol. XXXII., 1920.

If, as would appear legitimate, we can apply similar reasoning to a radiometer vane, we shall have to imagine a flow of gas to take place similar to that indicated in Fig. 3. On the cold side B, the gas will be flowing outwards from the centre, with consequent reduction of pressure on the vane, whilst on

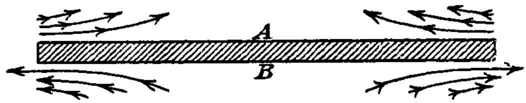

FIG. 3.—FLOW AROUND RADIOMETER VANE.
A is Hot Surface, *B* is Cold Surface.

the hot side A it will be flowing inwards towards the centre, with consequent increase of pressure on the vane. When the gas pressure is fairly high, and when the isotherms have consequently not departed much from the shapes indicated in Fig. 1, the flow of gas and the regions of excess and defect of pressure will be restricted to the peripheral portions of the disc. When,

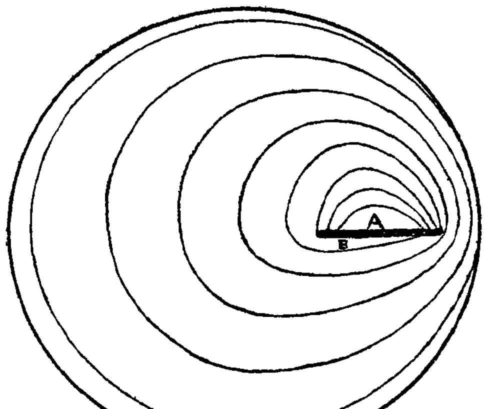

FIG. 4.—ISOTHERMS AROUND RADIOMETER VANE IN SPHERICAL GLASS BULB—MEDIUM GAS PRESSURE.*
A is Hot Surface, *B* is Cold Surface.

however, the gas pressure is lowered, the area over which the excess or defect of pressure is felt, will extend towards the centre, until at last the whole surface of the disc will be covered.

The shapes of the isotherms surrounding an actual radiometer vane enclosed in a spherical bulb are, of course, again

* The suggestions made by Mr. F. J. W. Whipple in the discussion are embodied in this figure.

far too complicated to calculate. From general considerations, however, it is possible to give a rough idea of their shapes. In practice, one side of the vane, although much cooler than the other, is still slightly above the temperature of the glass walls. and under such conditions Fig. 4 might represent sufficiently well the forms the isothermals would take at a medium gas pressure. An indication is also given in Fig. 5 of the directions in which the gas currents would flow. It will be seen that on both sides of the vane they flow towards the central region, but that on the hot side the flow is much more vigorous owing to the greater temperature gradient. It will be noticed further that the flow is greatest on the edge of the disc nearest the

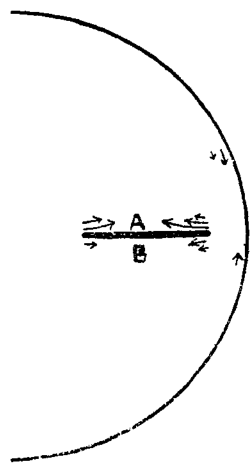

FIG. 5.—FLOW AROUND RADIOMETER VANE IN SPHERICAL GLASS BULB. *A* is Hot Surface, *B* is Cold Surface

glass, and also that in the gas layer on the glass itself there are two regions each capable of producing excess pressures—the most effective region being near the hot side of the disc.

Briefly, therefore, the modification introduced by the present explanation consists chiefly in a difference in the temperature ascribed to the surface layer of gas covering the vane, and in a consequent difference in the regions over which the flow of gas is supposed to take place. The question naturally arises, therefore, as to whether there are any radiometric phenomena that support the present view. For a reply we must turn our attention to the Crookes radiometer with slanting vanes.

In this instrument, a plan of which is shown in Fig. 6, a number of vanes are placed at an angle to the supporting radial arms. In one instrument the vanes were made of aluminium foil bright on both sides, and when radiation was allowed to fall upon them, vigorous rotation ensued (indicated in Fig. 6 by the arrow), and that in spite of an almost negligible difference in temperature of the two sides of the vanes. Crookes points out * that had the vanes pointed radially there would have been practically no tendency to rotation.

On Sutherland's theory, no very obvious explanation of this phenomenon can be given, but on the present theory this is at once possible.

The surfaces of the vanes will be covered on both sides by a gas layer whose temperature will rise from the edge towards

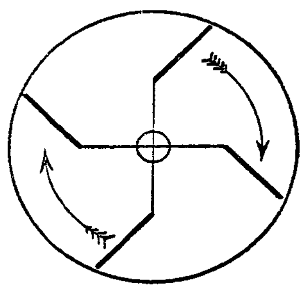

FIG. 6.—CROOKES RADIOMETER WITH SLANTING VANES.

the central regions. The gradient will be steepest on the part of the disc nearest the bulb, and here also the flow of gas will be most vigorous. We have to remember however that the gas currents can only flow as a result of a tangential reaction exerted on the material of the vane. Hence on both its surfaces we shall have outward tangential forces, and such forces will be capable of producing rotation. The side of the vane nearest the glass, moreover, will experience a pressure resulting from the difficulty the surface currents find in escaping, but this pressure will not in general be as important as the tangential forces.

The radiometer with slanting vanes connects itself naturally with the recent experiments of the author† on the edgewise movements of strips of foil placed at an angle to a glass

* Phil. Trans., 169, p. 282, 1878.

† Proc. Phys. Soc., Vol. XXXII., 1920.

wall, and exposed to the radiation from a lamp. It will be remembered that such strips showed edgewise movements towards the glass which completely masked the normal repulsion effect.

Let us now pass to the consideration of the radiometer constructed of hemispherical cups, mounted anemometer fashion. Sutherland says: "Consider a convex vane irradiated by a source on the normal through its middle point; then as the amount of heat that a surface absorbs depends on its obliquity to the incident radiation, the farther a part of the convex surface is from the middle, the less is it directly heated, and thus there is a continuous fall of temperature from the centre of the surface to the edge; conduction, if allowed time, tends to reduce the amount of fall but does not obliterate it, and conduction also establishes a fall of temperature along the back from the centre to the edge. Now the traction of the gas on the solid, is from hot to cold, so that both on the front and back of the vane there is a traction from centre to edge, whose resultant effect is to drag the vane away from the light when the vane is convex to it, so that the light appears to repel a convex surface. When the surface is concave the same reasoning applies, the gas exerts a traction from centre to edge, and therefore the light appears to attract it."

According to the above, therefore, the temperature gradient is made to depend on the changing obliquity of the surface of the cup. Rotation still takes place, however, when the obliquity is kept constant by replacing the hemispheres by hollow cones. It is true that, owing to a spreading of the stream lines more heat will here be conducted away from the peripheral regions of the cone than from other regions, but with cups of good conducting material, such as aluminium, the temperature gradient so produced would be very small. Previous experiments by the author have demonstrated, moreover, that such temperature gradients are not of much importance. For the rotation of the conical cups, therefore, Sutherland's theory gives no easy explanation.

If we realise, however, that a rarefied gas does not necessarily take the temperature of the surface with which it is in contact, and if we imagine, as is legitimate, that a temperature gradient, in the gas layer, extends from the periphery of the cone towards the apex, there is no difficulty in modifying Sutherland's original explanation to suit this new case. Such a modification will resemble that given for the radiometer with slanting vanes.

In his Paper Sutherland deals with other experiments performed by Crookes, but the differences that have to be introduced into his explanations on the present theory are so small that further discussion is not necessary.

It might be said, therefore, when all the facts that have been discussed are taken into consideration, that Sutherland's theory, as it stands, is apparently not capable of explaining certain radiometric phenomena, but that when modifications, supported by modern experimental knowledge, are made in the thermal surface conditions, such explanations become at once possible.

ABSTRACT.

The Paper gives a short account of a theory of the Crookes Radiometer worked out by Sutherland in 1896, but unfortunately since much neglected. The theory as it stands will not explain many radiometric phenomena, but it is shown that when modifications, depending on the modern knowledge of thermal surface conditions, are made, such explanations become possible.

Radiometer action, especially at the higher gas pressures, would appear to depend essentially on the formation of gas currents near the radiometer vane. These currents are distinct from convection currents, but are closely connected with the phenomena of thermal transpiration.

The present work is the natural outcome of previous work by the author (Proc. Phys. Soc., Vol. XXXII.).

DISCUSSION.

Mr. F. J. W. Whipple pointed out that as the isothermals were drawn in Fig. 4, they indicated that the colder side of the vane was actually colder than the walls of the bulb, whereas actually it was hotter. He indicated alterations in the shape of the isothermals which would correct this. In connection with the general presentation of the problem he would have liked it better if it had been based more on kinetic theory and less on hydrodynamical flow. It was necessary to introduce kinetic theory to explain the difference of pressure between the hot and cold ends of a tube. When the mean free path was large, a particle entering one end of the tube would strike the sides tangentially and pass through the tube. When equilibrium was reached the number of particles entering at each end must be equal, so that $\rho v=\rho' v'$. But the condition for equality of pressure was $\rho v^2=\rho' v'^2$.

Mr. West, in reply, said he was not quite clear at the moment on the point raised about Fig. 4. He would look into it and communicate a reply.* In an earlier Paper he had pointed out what Mr. Whipple had just mentioned, that at low pressures it was the conservation of matter that determined equilibrium, while at high pressures it was the equality of pressures.

* See footnote to page 227.